sweep generator as part of its automatic-levelling control facilities.

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MAXIMALLY FLAT ATTENUATION AND DELAY CHARACTERISTICS

In a recent letter,[1] a class of polynomial transfer functions, intermediate between the Butterworth and the Thomson families, has been defined. The author mentions in the conclusion that, for each member of the class, the stability (occurrence of r.h.p. poles) should be investigated. This study has already been completed up to the ninth degree by Golay.[2] It happens that most of the characteristics are unstable, with the exception of the following cases: one condition on the phase and two conditions on the attenuation, and one condition on the attenuation and all the other conditions on the phase. Furthermore, these stable characteristics are rather unsatisfactory; despite the Taylor approximation around the origin, the attenuation ripple in the band is rather high, owing to an undesirable amplitude rise at the bandedge. From a purely theoretical standpoint, the class of investigated functions seems to be almost void; this stresses the danger of a mixed set of conditions on both amplitude and phase.

From a more general point of view, the practice of imposing simultaneous conditions on amplitude and phase deserves to be questioned; most of the time, they originate from a pulse-transmission filtering problem, in which some requirements are put on the transient behaviour, such as the risetime and the overshoot of the step response. These constraints can be met in a rigorous fashion by using the filter family defined by Schüssler *et al.*;[3] it is difficult to conceive what can by gained by transposing requirements from the time domain into the frequency domain.

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CONSTANT-CURRENT SOURCE WITH UNUSUALLY HIGH INTERNAL RESISTANCE AND GOOD TEMPERATURE STABILITY

A transistor operating at 1 mA, with 10 kΩ in the emitter lead, gives a collector a.c. output resistance in the region of 10 MΩ. A 2-transistor circuit is described, having an output resistance of approximately 50 MΩ, with much improved temperature stability. A further modification gives an output resistance of about 1000 MΩ, corresponding to an 'aiming voltage' of 1 MV.

The well known circuit of Fig. 1*a*, operating at 1 mA d.c. with $R_e = 10\,\mathrm{k}\Omega$, has been found to have an a.c. output resistance typically in the region of 10 MΩ. The output resistance is not infinite, because

(*a*) the base current of a transistor at constant emitter current falls with increasing collector voltage, causing an equal and opposite increase in collector current
(*b*) the base–emitter voltage, at almost constant emitter current, falls with increasing collector voltage; with the base fixed in potential, this voltage variation appears as an increase in the voltage across R_e, causing a corresponding small increase in emitter current.

In practice, effect (*a*) is usually the dominant one with this circuit. The magnitude of the two effects can be predicted from the collector-current/collector-voltage family of curves for the transistor used; or, alternatively, a formula may be derived using an equivalent circuit for the transistor, the hybrid-Π circuit being preferred.[1]

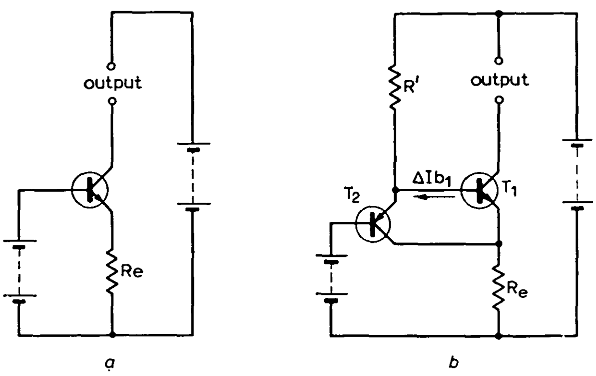

Fig. 1
a Simple constant-current source
b Improved constant-current source

A further shortcoming of the circuit of Fig. 1*a* is that the collector–base capacitance of the transistor appears across the output, reducing the output impedance at high frequencies. Also, if the circuit is used to charge a capacitor, for the purpose of generating a linear ramp, the voltage-dependent nature of this collector–base capacitance will result in nonlinearity of the ramp.

The output current of the circuit is dependent on ambient temperature, because of variation with temperature of the following properties:

(i) transistor emitter–base voltage
(ii) transistor current gain
(iii) leakage current

The circuit shown in Fig. 1*b* is largely free from the above defects.

When the T_1 collector voltage increases, the base current becomes less, as in the previous circuit. In the new circuit, however, nearly all of this change in base current ΔI_{b1} flows to the emitter of T_2, giving rise to an almost equal increase in T_2 collector current. This current increment flows to T_1 emitter, resulting in a reduction of the T_1 emitter current by very nearly the amount necessary for preventing the increase in the T_1 collector current which would otherwise occur. An a.c. output resistance of 50 MΩ is readily achieved with this circuit at $I_{c1} = 1\,\mathrm{mA}$ and $R_e = 10\,\mathrm{k}\Omega$.

It is now mechanism (*b*) above that is the main effect limiting the output resistance; this is considered in more detail later. The circuit possesses the following further advantages:

(*a*) Assuming T_1 and T_2 to be of similar construction, e.g. both silicon planar, the effects of ambient temperature on the emitter–base voltages of the two transistors will largely balance out, as far as the voltage across the main current-determining resistor R_e is concerned. If we assume a coefficient of 0·25 mV/degC for the two transistors taken together, and a 10 V drop across R_e, the output-current variation due to this cause will be 0·0025% per degC. This is likely to be a negative coefficient, since the coefficient for a single transistor tends to become numerically greater as the working current is reduced.[2] (The emitter–base-voltage variations of T_2 will nevertheless appear across R', causing a current change in this resistor, which finds its way through T_2 to the emitter and collector of T_1. This current change will be quite small, however, R' being much larger than R_e in value. If we assume 10V across R', and the working current of T_2 to be 10% of that of T_1, a 2·5mV/degC variation in T_2 emitter–base voltage will cause T_1 collector current to change by −0·0025% per degC.)

(*b*) Current flowing in the collector–base capacitance of T_1 is transmitted via T_2 back to T_1 emitter, causing a collector-current variation in T_1 such as will almost balance out the flow of capacitive current in the lead to the output terminals. Effective output capacitances of well under 1 pF are obtained.

(*c*) Variation of the current gain of T_1 with temperature causes I_{b1} to vary; but the action of T_2 almost prevents the collector current of T_1 from being affected by this—just as already described in connection with the changes in I_{b1} caused by output-voltage variations. An effect for which the circuit does not provide any compensating action is variation of T_2 current gain, and hence base current, with temperature However, because T_2 is operated at only 5 or 10% of the current of T_1, this effect is very small. (If we assume the common-emitter current gain α' of T_1 to increase by 1% per degC, which is a typical value, and α' is taken as 100, and if T_2 operates at 10% of the current of T_1, the variation in T_1 output current due to this cause will be only −0·001% per degC.)

(*d*) Variation with temperature of the leakage current I_{CBO} of transistor T_1 is prevented from affecting the output current by the same compensating action as has already been described. Whereas any variation in I_{CBO} of T_2 affects the output current in full measure, the value of I_{CBO} in this transistor will be very much smaller than normal, because the transistor is operated with zero voltage between the base and the collector. Thus the overall effect of leakage currents, assuming silicon transistors, should be negligibly small.

In an experimental version of the circuit of Fig. 1*b*, using types 2N2926 ($\alpha' \simeq 115$) for T_1 and BFY64 for T_2, an output impedance of about 60MΩ was obtained at frequencies below 1kHz at an output current of 1mA. For ramp-generating applications, this corresponds to a so called 'aiming voltage' of 60kV.

While the above performance figures are probably good enough for any practical application, it was thought interesting to investigate how high an a.c. output resistance could be obtained if R_e in Fig. 1*b* were replaced by a constant-current circuit of the type in Fig. 1*a*, thus preventing the small variation in the emitter voltage of T_1 that occurs when the collector voltage changes from affecting the current in T_1. With this modification, an a.c. output resistance of about 1000MΩ was obtained, corresponding to an aiming voltage of 1 MV. This modification, of course, causes the very desirable temperature-compensating features of the straightforward circuit of Fig. 1*b* to be lost, but further elaboration would obviously enable these virtues to be regained if desired.

The arrangement used for measuring the a.c. output impedance of these circuits is shown in Fig. 2. During these experiments, it was found necessary to insert a 220Ω resistor in the base lead of T_1 (2N2926), with a 1000pF capacitor between the base and the emitter, to prevent a tendency for a r.f. parasitic oscillation to occur.

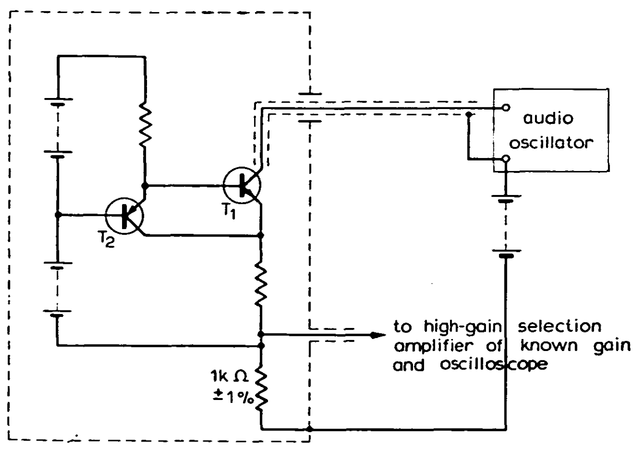

Fig. 2 *Circuit for measuring the a.c. output impedance of a constant-current source*

Another application for circuits of the type described in this letter is in the production of a sinewave-signal source of very low harmonic distortion and very high impedance. In the circuit of Fig. 1*a*, with a signal voltage applied in series with the base, the distortion in the collector current is liable to exceed that of the current flowing in R_e, because of the variable proportioning of the emitter current between base and collector during an a.c. cycle. The circuit of Fig. 1*b* is particularly effective in overcoming this difficulty, since variation in the proportioning of emitter current in T_1 between base and collector is compensated by the action of T_2, as already described, and T_2 operates with virtually constant interelectrode voltages. The circuit of Fig. 1*b* was, in fact, first investigated because of its ability to give very low distortion, which was further reduced by using the voltage across R_e to provide negative feedback to an earlier stage of an amplifier system. Distortion figures of less than 0·01% were obtained at a high output level.

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COUPLED-WAVE EQUATIONS FOR PROPAGATION IN STRATIFIED COMPRESSIBLE MAGNETOPLASMAS

This letter deals with the propagation of coupled electromagnetic and electroacoustic waves in a magnetised compressible continuously stratified electron plasma. Some first-order coupled-wave equations are derived.

Considerable attention has recently been given in the literature to the propagation and coupling of electromagnetic and electron-acoustic waves in compressible plasmas. The basic equations have been studied, particularly by Chen and Cheng, and various boundary-value problems have been investigated, particularly by Seshadri and Wait. The papers produced by these authors are too numerous to list here. In most of this work, the plasma has been taken to be homogeneous, or to consist of homogeneous regions with sharp boundaries. In a region of homogeneous nonmagnetised electron plasma, electromagnetic and electron-acoustic waves propagate independently. Coupling between the two wave types occurs at a boundary between regions if the electric vector has a component normal to the boundary; coupling can also occur in a continuously varying inhomogeneous region.

Approximate studies of fields in continuously stratified compressible plasmas have been made by a number of authors.[1–4] Very recently, several authors have considered the problem of obtaining rigorous wave equations describing propagation in inhomogeneous compressible plasmas.[5–10] In particular, some first-order coupled-wave equations have been derived for propagation in stratified electron plasmas.[6, 10] In the present letter, the work of Burman[10] is extended to allow for the effect of a magnetostatic field.

The model of the plasma taken here consists of electrons neutralised by positive ions; the motion of positive ions in the electromagnetic field is neglected. The waves are described by Maxwell's equations coupled to the single-fluid equations of hydrodynamics; and the amplitude of the waves is taken to be sufficiently small that linearised equations are applicable. A time factor $e^{j\omega t}$ is taken, where ω is the angular frequency of the fields and t is the time. The permeability of the plasma has the free-space value μ_0, ϵ_0 is the permittivity of free space and e is the charge on an electron (a negative quantity). The electrons have number density $N_0 + N$, where N_0 is the