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FEEDBACK-GENERATED PHASE MODULATION IN AUDIO AMPLIFIERS

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FEEDBACK-GENERATED PHASE MODULATION IN AUDIO AMPLIFIERS

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ABSTRACT

It is shown that by the application of feedback, an arbitrary open-loop amplitude nonlinearity of an audio amplifier is converted into a corresponding closed-loop phase nonlinearity. In the standard case, the effect manifests itself by the amplitude of the audio signal phase-modulating the high-frequency components of the signal.

I. INTRODUCTION

An idealized theoretical model of a feedback audio amplifier is assumed to have the following characteristics:

I.1. The forward path has perfectly linear, time-invariant, frequency-independent, memoryless open-loop gain

$$A = \prod_{i=1}^{n} A_i \qquad (1)$$

which is the product of n arbitrarily distributed individual stage gains A_i .

I.2. Associated with the amplifier stages are k arbitrarily nonlinear but continuous, memoryless, frequency-independent nonlinearities $e_i(x_i)$, arbitrarily distributed in the amplifier circuitry, and dependent on signal amplitude x_i only. The open-loop transfer characteristic can then be written as

$$F(x) = A \left[1 + \prod_{i=1}^{k} e_i(x_i) \right] \qquad (2)$$

I.3. The feedback stability constraints require that the forward path has a single, time-invariant, signal-independent pole ω_0 , situated anywhere in the forward path. The open-loop transfer function of the amplifier is then

$$F(s) = \frac{\omega_0}{s + \omega_0} A \qquad (3)$$

where s is the Laplace operator.

The total open-loop transfer equation then becomes

$$F(x,s) = \frac{\omega_0}{s+\omega_0} \left[1 + \prod_{i=1}^{K} e_i(x_i) \right] \prod_{i=1}^{n} A_i \quad . \qquad (4)$$

This model is sufficiently general to cover most cases of practical interest, and sufficiently simple to allow explicit illustration of the effect to be discussed.

2. ANALYSIS

The feedback path is assumed to be purely resistive, time-invariant, perfectly linear, and to have a gain of β. By denoting

$$\varepsilon(x) = \prod_{i=1}^{n} e_i(x_i) \qquad (5)$$

the closed-loop transfer equation becomes

$$F_c(x,s) = \frac{A\left[1+\varepsilon(x)\right] \omega_0 (s+\omega_0)^{-1}}{1+\beta A\left[1+\varepsilon(x)\right] \omega_0 (s+\omega_0)^{-1}} \qquad (6)$$

which can be factored into

$$F_c(x,s) = \frac{A\left[1+\varepsilon(x)\right]}{1+\beta A\left[1+\varepsilon(x)\right]} \cdot \frac{1}{1+\frac{s}{\omega_0}\left\{1+\beta A\left[1+\varepsilon(x)\right]\right\}^{-1}} \qquad (7)$$

The first part is the conventional form of the time-independent feedback equation and describes the low-frequency closed-loop gain of the amplifier. It illustrates how feedback reduces nonlinearity at frequencies below the dominant pole ω_0 .

The second part is complex and describes the closed-loop frequency response of the amplifier. It is important to notice that the closed-loop cut-off frequency of the amplifier now becomes

$$\omega_c = \omega_0 \left\{1+\beta A\left[1+\varepsilon(x)\right]\right\} \qquad (8)$$

and is seen to be a function of the signal amplitude x through the open-loop nonlinearity $\varepsilon(x)$.

3. CLOSED-LOOP PHASE CHARACTERISTIC

Replacing $s=j\omega$, and assuming that the open-loop nonlinearity is small, $\varepsilon(x) \ll 1$, and that feedback is large, $(1+\beta A) \gg 1$, the phase angle of Eq.(7) is

$$\phi_c(x,s) = -\arctan\left\{ \frac{\omega}{\omega_{co}} \left[1-\varepsilon(x)\right] \right\} \qquad (9)$$

where $\omega_{co} = \omega_0(1+\beta A)$, i.e., the steady-state closed-loop cut-off frequency when $\varepsilon(x)=0$.

Subtracting Eq.*(9)* from the fixed part of the phase response, i.e., Eq.*(9)* at $\varepsilon(x)=0$, and by denoting

$$K = \frac{\omega}{\omega_{co}}\{1+[\frac{\omega}{\omega_{co}}]^2\}^{-1} \tag{10}$$

as a frequency-dependent scalar coefficient, the characteristic amplitude-to-phase transform equation can be found to be

$$\Phi(x) = -\arctan K\varepsilon(x) \tag{11}$$

As can be seen from Eq.*(11)*, any open-loop amplitude nonlinearity $\varepsilon(x)$ will by the application of feedback be directly transformed into a corresponding phase nonlinearity $-\arctan K\varepsilon(x)$. It can also be noted that the law of nonlinearity $\varepsilon(x)$ is preserved in this mapping.

The above analysis has concentrated on the phase nonlinearity only. As is evident from Eq.*(7)*, also amplitude nonlinearity is present. However, since it has been extensively discussed in classical feedback theory, it will not be analyzed here.

4. PHASE MODULATION

To crudely illustrate the effect discussed in Chapters 2 and 3, the amplifier is subjected to a two-tone SMPTE-type intermodulation test signal, consisting of a low-frequency signal $B_1\sin\omega_1 t$ and a high-frequency signal $B_2\sin\omega_2 t$.

Assuming $B_1 \gg B_2$, $\omega_2 \gg \omega_1$, $1+\beta A \gg 1$,
$\varepsilon(B_1) \ll 1$, $\omega_0 \gg \omega_1$, $\omega_c \gg \omega_2$,

Eq.*(7)* gives the ω_2 component of the output signal as

$$B_2' = A_{cl}B_2\sin\{\omega_2 t - \frac{\omega_2}{\omega_{co}}[1-\varepsilon(B_1\sin\omega_1 t)]\}, \tag{12}$$

where A_{cl} is the amplifier closed-loop gain, i.e., the first part of Eq.*(7)*.

For illustration, assume that $\varepsilon(x)=x$, i.e., distortion is of pure second order. Eq.*(12)* can then be developed to

$$B_2' = A_{cl}B_2\{\sin(\omega_2 t)[J_0(m) + 2\sum_{k=1}^{\infty} J_{2k}(m)\cos(2k\omega_1 t)] + 2\cos(\omega_2 t)\sum_{k=0}^{\infty} J_{2k+1}(m)\sin[(2k+1)\omega_1 t]\}, \tag{13}$$

where $J_k(m)$ is Bessel function of integer order k,
and $m = \frac{\omega_2}{\omega_{co}}B_1$ is a modulation index.

This equation is seen to be the classical representation of a phase-modulated wave with carrier frequency ω_2 and side-bands $\omega_2 \pm k\omega_1$. Feedback has thus converted the open-loop amplitude modulation described by Eq.*(4)* into corresponding phase modulation described by Eq.*(13)*.

5. DISCUSSION

It has been shown that
the application of feedback, while performing as advertised in reducing amplitude nonlinearities, also maps any open-loop amplitude nonlinearity into a corresponding closed-loop phase nonlinearity.
It has also been illustrated that
this transformation converts any open-loop amplitude intermodulation distortion into equivalent closed-loop phase intermodulation distortion.

The analysis has been derived under the assumption that the open-loop amplitude distortion is small. In practical high-feedback amplifiers this assumption is not necessarily valid, and the transform equation Eq.*(11)* cannot be obtained in closed form.

The assumption that the dominant pole ω_0 is signal-independent may also not be true in many high-feedback amplifier topologies. However, in both of the above cases, the effect discussed can be expected to be aggravated from the results obtained here.

It should be noted that TIM (transient intermodulation distortion, also known as SID, slewing induced distortion) can be considered as a limit case of this effect, as discussed elsewhere[1].

By the very nature of the effect, it is understandable that many of the present measurement methods, notably the SMPTE-IM and the THD, are incapable of detecting the effect.

The audibility of this effect, a measurement method for it, and the ways to eliminate or minimize it,will be subjects to later papers. Based on preliminary results, the effect seems to be particularly annoying in amplifiers using high values of feedback to suppress cross-over distortion, which causes a relatively large and abrupt momentary change in the open-loop transfer characteristic.

REFERENCE

[1] M. Otala and E. Leinonen,"Possible methods for the Measurement of Transient Intermodulation Distortion". 53rd Convention of the Audio Eng. Soc., Zürich, Switzerland, March 1976. Preprint available from the Technical Research Centre of Finland, Report 16/1976, Electrical and Nuclear Technology Series. 16 p.